A THREE CORNER HAT-BASED ANALYSIS OF STATION POSITION TIME SERIES FOR THE ASSESSMENT OF INTER-TECHNIQUE PRECISION AT ITRF CO-LOCATED SITES

ABSTRACT REFERENCE NUMBER : 1490763
PAPER NUMBER : G53B-1142

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1. INTRODUCTION

- Three corner hat (TCH) has been applied to the station position time series of the 4 space-geodetic (SG) techniques (VLBI, GPS, SLR, DORIS) defining the International Terrestrial Reference Frame (ITRF)
- As an alternative to statistics defining the ***intrinsic precision*** (e.g. repeatability, standard deviation), TCH can be used in this context to infer the ***relative/inter-technique precision*** of station positions through the comparison of the performances of the 4 SG techniques
- If station position time series are acquired from at least 3 co-located SG techniques, the TCH can provide the relative precision of each station included in the co-located site.
- In this study, adopting the data set used for the definition of the current ITRF2008, *(i)* we ***assess*** via the TCH the *relative* precisions each ITRF co-located site with a sufficiently adequate observing history, *(ii)* we ***compare*** at each site *relative* vs *intrinsic* precision (i.e. repeatability of geodetic positioning, standard deviation derived from SINEX files)

2. THREE CORNER HAT

$$x_i(t)=s(t)+w_i(t) \quad i=VLBI, SLR, GPS, DORIS$$

- $\boldsymbol{x_i}$ identifies the measured value (i.e, the station position as determined by the i-th technique and expressed in a local geodetic reference frame), ***s*** indicates the ***geophysical signal***, ***i*** identifies the space-geodetic technique, $\boldsymbol{w_i}$ accounts for both the measurement and systematic errors affecting the *i-th* technique
- The noise processes $\boldsymbol{w_i}$ are assumed to be statistically uncorrelated one to another and independent
- We assume that each co-located technique senses the same geophysical signal ***s***
- This way, the pair-wise difference among the measurements eliminates the common signal ***s*** and uniquely reflects the differences between the measurement errors of the two techniques
- Under these assumptions, from the evaluation of the empirical variance of the difference process $(\mathbf{x_i}\text{-}\mathbf{x_j})$ we can compute the variance of the noise process $\mathbf{w_i}$ associated with the i-th technique at the co-located site
- In order for the difference process $(\mathbf{x_i}\text{-}\mathbf{x_j})$ to be rigorously defined, station position time series of VLBI, GPS, SLR and DORIS have to be *(i)* expressed in the same reference frame and *(ii)* aligned in time

3. DATA SETS used

- The entire set of SINEX files submitted by the IVS, IGS, ILRS and IDS for the computation of ITRF2008 has been analysed in this investigation (see *Table 1*):

(i) ***Weekly SINEX*** *files* of station positions for GPS, SLR, and DORIS

(ii) ***Daily SINEX*** *files of datum-free normal* equations for VLBI

- Table 1 shows the time span, the kind of solution provided by the 4 official technique services (IVS, IGS, ILRS and IDS) and the number of sinex files considered for each technique in the analysis

Table 1. *Time span, kind of solution provided by IVS, IGS, ILRS and IDS, constraints applied to the solutions and total number of SINEX files considered for each technique.*

Technique	Data Span	Solution	Constraints	# SNX
VLBI	1980 – 2009	Normal Equations	Datum Free	3658
GPS	1997 – 2009	Variance - covariance	Minimal	653
SLR	1983 - 2009	Variance - covariance	Loose	1041
DORIS	1993 - 2009	Variance – covariance	Minimal	830

4. DATA ANALYSIS

- ***Data Editing.*** SINEX files have been cleaned with the aim of removing outliers. The cleaning relies on the stacking of SINEX files for each of the 4 techniques. Outliers w.r.t. a linear model (*i.e.* stacked reference frame) have been removed from the SINEX files.
- ***Transformation in ITRF2008.*** Translations and rotations between each single technique SINEX file and the official ITRF2008 have been estimated. All the SG solutions have been transformed into ITRF2008 applying the estimated rotations and translations. This way, the SG solutions have been consistently expressed w.r.t. the same reference frame and can be therefore inter-compared.
- ***Extraction of time series at co-located sites.*** Time series of station positions have been extracted at ITRF co-located sites. GPS, SLR and DORIS provide time series at a weekly resolution, whereas VLBI time series are daily. Time series at co-located sites have been de-trended, removing piecewise linear trends. The station position discontinuities identified for the ITRF2008 computation have been used for the piecewise linear regressions (see *Fig 1*).
- ***Temporal alignment.*** Simultaneous observations among the 4 techniques have been selected. Prior to the temporal alignment, daily VLBI time series have been aggregated into weekly time series and linearly interpolated in order to make them comparable with GPS, SLR and DORIS solutions.
- ***TCH application.*** Once de-trended and temporally aligned, the co-located time series can be differentiated. Difference processes have been formed (see *Fig 2*) and TCH-derived variances have been estimated (see *Table 2*).

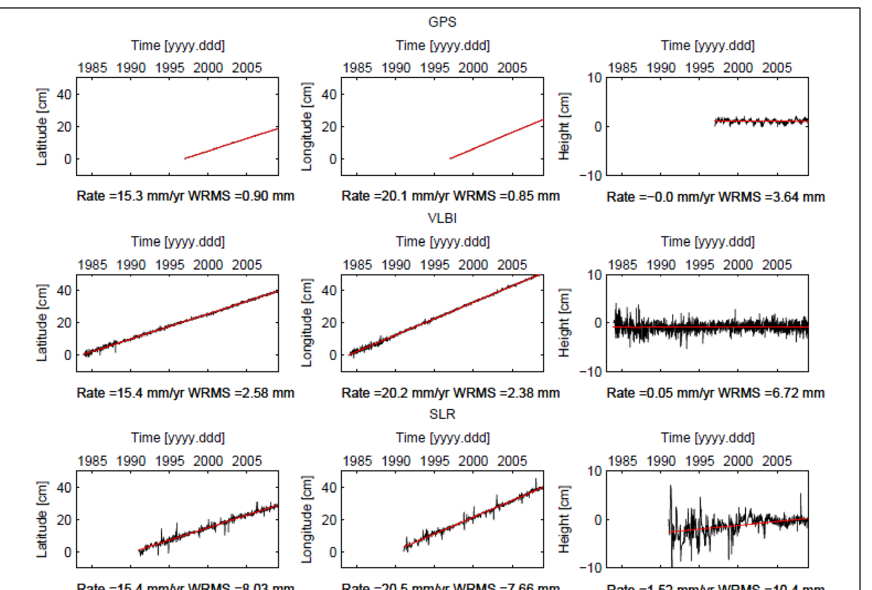

Figure 1. *Time Series of station positions for the SLR (8834), the GPS (WTZR) and the VLBI (7224) co-located at WETTZELL as obtained from the transformation in ITRF2008. Linear trend estimates and WRMS (about the linear trend) are reported for each technique and component.*

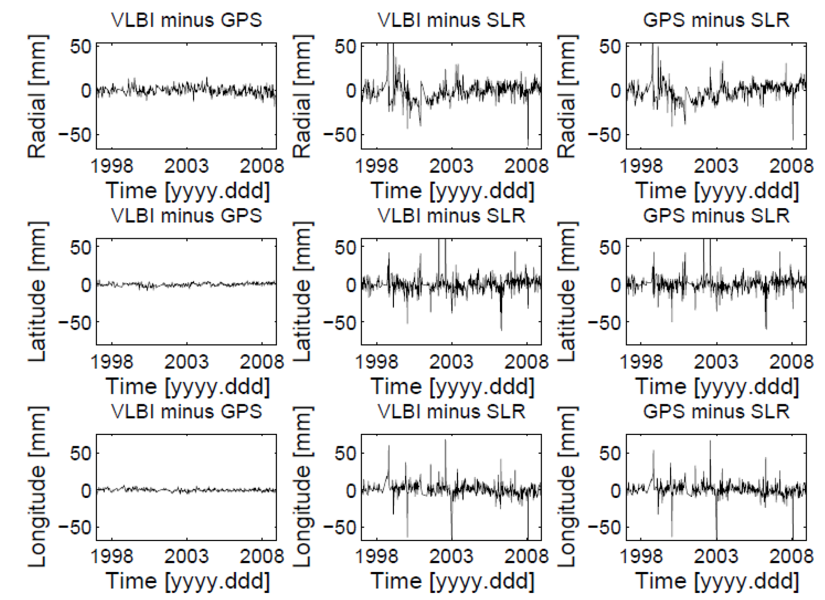

Figure 2. *Time Series of the difference processes for the SLR (8834), the GPS (WTZR) and the VLBI (7224) co-located at WETTZELL. These differences, computed after the linear trend has been removed from the mono-technique time series (see Figure 1), are the input to the TCH algorithm*

5. RESULTS and DISCUSSIONS

- ***19 ITRF co-located sites*** with number of simultaneous observations > 30 in the time window 1997-2009 have been selected for the TCH analysis (see *Table 2* for results).
- Output of the TCH are the variances of the station position time series for each technique computed on the (North, East, Height) components. Hartebeesthoek proved to be the only 4-way co-location with an adequately large number of simultaneous observations. In that case, the generalised TCH has been applied to infer the relative precision of the 4 SG techniques.
- Table 2 reports for each co-location *(i)* the σ obtained with the TCH, *(ii)* the WRMS (repeatability) computed on the same set of observations used for the TCH, *(iii)* the difference between WRMS and TCH-derived σ, *(iv)* the formal error extracted by the SINEX files transformed into ITRF2008, *(v)* the number of simultaneous observations among the co-located techniques. The WRMS is computed after the linear trend has been removed from each time series
- In principle, the higher the number of simultaneous observations, the more robust the TCH-derived sigmas (see sites marked in red in *Table 2)*
- When comparing columns **T** (TCH-derived sigmas) *and* **σ** (SNX-derived sigmas, i.e. formal errors) in *Table 2*, one can observe the formal errors are in general overly optimistic (σ<T): the ratio $(T/\sigma)^2$ can thus provide a scaling factor to be applied to the covariance matrix of the SG solutions reported in the SNX files (e.g. the values of T/σ for the height component of Hartebeesthoek are 5.3, 5.3, 2.6, 0.8 for VLBI, SLR, DORIS and GPS respectively)
- SG time series used in this study contain the seasonal signature due to non-tidal loading effects, which have not been removed during the SG data reduction. As a result, the WRMS (column **W** in *Table 2*) of the time series accounts for the seasonal variability and, in principle, it should be larger than the TCH-derived sigmas (column **T** in *Table 2*).
- The reduction (i.e. positive values of the difference **W-T**) due to the removal of seasonal signatures when computing the inter-technique difference processes is observable uniquely for the Height component of GPS (see rows **G** in the column **W-T for Wettzell, Greenbelt, Concepcion, Arequipa** *in Table 2*).

- GPS is characterised by the highest relative precisions (in the TCH sense) in all the three North, East and Height components, followed by VLBI, SLR and DORIS. GPS relative precisions are below the mm level in the horizontal components and attain values of a few mm in the Height component.

Table 2. *Results per components (North, East, Height) of the TCH analysis applied to the ITRF co-location sites.* **Tc** (technique) : **V** VLBI, **G** GPS, **D** DORIS, **S** SLR. **T** : TCH-derived **σ** in **mm**. **W :** repeatability in **mm**. **σ** : SINEX formal error in **mm**. **W-T :** difference between WRMS and TCH in **mm. O :** number of observations. Sites marked in red identify the co-locations with the largest number of simultaneous observations.

Sites	Tc	North					East					Height				
		T	W	W-T	σ	O	T	W	W-T	σ	O	T	W	W-T	σ	O
NY-ÅLESUND	V	1.7	1.8	0.1	0.4	352	1.5	1.4	-0.1	0.4	351	5.7	6.5	0.8	1.5	352
	G	0.6	1.0	0.4	1.2	352	1.1	0.9	-0.2	1.1	351	0.6	4.6	4.0	5.6	352
	D	9.3	8.4	-0.9	2.6	352	9.7	8.3	-1.5	2.4	351	12.3	10.7	-1.6	2.4	352
METSAHOVI	G	6.7	0.9	-5.8	1.3	75	5.7	0.9	-4.8	1.0	20	3.9	4.5	0.6	3.4	75
	S	23.6	16.8	-6.8	290.2	75	8.3	7.9	-0.4	10.8	20	18.9	12.3	-6.6	146.6	75
	D	10.5	10.1	-0.4	1.7	75	9.6	11.3	1.8	5.5	20	16.4	13.8	-2.6	2.3	75
MATERA	G	1.4	1.1	-0.3	0.8	231	1.7	0.9	-0.9	0.8	231	4.1	3.3	-0.7	2.8	229
	V	1.9	2.1	0.2	0.8	231	1.3	2.0	0.7	1.1	231	5.3	5.5	0.3	1.8	229
	S	16.3	11.3	-5.0	7.4	231	14.9	9.8	-5.0	12.2	231	13.9	8.6	-5.3	2.0	229
WETTZELL	G	0.4	0.9	0.5	0.8	440	0.3	0.8	0.5	0.9	253	2.2	3.7	1.4	3.2	496
	V	2.1	1.9	-0.1	0.7	440	1.5	1.6	0.1	1.3	253	4.0	4.5	0.5	1.8	496
	S	11.3	7.0	-4.3	3.9	440	8.9	6.1	-2.8	5.6	253	10.0	7.3	-2.7	1.0	496
SHANGHAI	G	0.9	1.8	0.9	2.4	40	1.4	1.2	-0.2	3.5	30	3.9	4.9	1.0	9.5	50
	V	2.2	2.7	0.5	1.4	40	3.7	4.1	0.4	1.1	30	7.6	11.1	3.5	2.6	50
	S	16.4	11.3	-5.1	6.1	40	19.9	11.5	-8.4	4.9	30	20.6	11.8	-8.7	4.9	50
ST JOHN'S	V	5.9	4.3	-1.6	3.7	25	3.0	4.0	1.0	2.1	25	24.1	21.1	-3.0	6.9	25
	G	2.2	0.9	-1.4	0.9	25	2.2	0.9	-1.3	0.9	25	4.0	2.2	-1.9	2.7	25
	D	5.3	5.5	0.3	2.2	25	14.5	13.8	-0.7	3.8	25	9.6	11.8	2.2	2.7	25
YELLOWKNIFE	V	1.9	2.9	1.0	2.4	39	3.3	3.2	-0.1	1.1	35	6.4	6.6	0.2	4.6	23
	G	2.8	0.9	-1.9	0.8	39	0.6	0.6	0.0	0.8	35	6.4	4.8	-1.7	2.2	23
	D	8.5	7.3	-1.2	2.0	39	20.5	14.9	-5.6	3.0	35	10.4	9.9	-0.5	2.6	23
FAIRBANKS	V	3.5	2.8	-0.6	1.9	360	2.6	2.3	-0.4	2.5	359	6.0	6.1	0.1	3.8	359
	G	2.2	2.4	0.2	1.7	360	0.8	1.4	0.6	1.3	359	4.9	5.6	0.7	5.1	359
	D	11.0	10.7	-0.3	5.1	360	12.9	12.9	0.0	7.7	359	12.9	13.1	0.2	6.2	359
KAUAI	V	2.8	3.2	0.4	2.0	538	3.1	3.3	0.2	2.1	538	6.4	6.1	-0.2	2.7	538
	G	2.0	1.2	-0.8	1.1	538	2.1	1.3	-0.8	1.2	538	4.2	4.0	-0.2	3.7	538
	D	11.3	9.4	-1.8	3.1	538	17.2	14.1	-3.1	5.9	538	14.8	12.7	-2.1	4.5	538
FORT DAVIS	G	0.6	1.0	0.4	1.2	30	1.1	1.1	0.0	1.1	30	4.6	3.6	-1.0	2.9	77
	V	6.3	2.1	-4.2	0.3	30	6.9	1.5	-5.4	0.3	30	7.1	7.2	0.1	0.7	77
	S	1.9	5.7	3.8	5.1	30	1.4	6.8	5.4	3.9	30	7.6	6.7	-0.9	0.7	77
GREENBELT	G	2.3	1.4	-0.8	0.9	323	2.4	0.9	-1.5	0.9	323	2.6	3.7	1.2	3.1	323
	S	8.0	5.7	-2.3	4.0	323	8.7	5.5	-3.2	3.5	323	9.0	4.6	-4.5	1.2	323
	D	11.8	10.1	-1.8	2.4	323	13.7	13.0	-0.7	4.6	323	14.1	12.3	-1.8	3.4	323
MONUMENT PEAK	G	2.2	1.3	-0.8	1.7	90	1.5	1.4	-0.1	1.2	30	2.1	2.2	0.1	3.8	106
	S	6.6	5.0	-1.6	16.1	90	4.5	4.1	-0.4	6.2	30	5.9	3.8	-2.1	1.6	106
	D	7.1	7.7	0.6	2.4	90	9.6	9.9	0.2	4.3	30	7.9	7.9	0.0	5.9	106
CONCEPCION	G	1.5	1.7	0.2	1.1	197	2.4	1.4	-1.0	1.1	197	3.6	5.1	1.5	3.4	196
	V	4.6	4.6	0.0	2.5	197	3.1	3.9	0.8	1.9	197	10.6	10.1	-0.5	9.6	196
	S	13.9	10.2	-3.7	10.1	197	17.1	13.1	-4.0	18.0	197	11.1	7.4	-3.7	1.1	196
AREQUIPA	G	8.7	6.2	-2.5	0.9	168	11.0	10.8	-0.2	1.1	164	3.2	6.3	3.1	3.2	167
	S	11.2	10.2	-1.0	21.4	168	18.9	10.9	-8.0	18.0	164	10.5	8.0	-2.5	8.1	167
	D	16.2	13.2	-3.0	3.9	168	22.1	18.8	-3.3	9.3	164	20.5	16.7	-3.8	6.1	167
TIDBINBILLA	G	8.3	1.5	-6.8	1.5	62	3.5	1.5	-2.0	1.4	45	3.9	4.0	0.0	3.5	55
	S	12.0	12.4	0.4	20.9	62	6.7	7.5	0.8	8.0	45	8.9	9.1	0.2	0.8	55
	D	14.7	16.3	1.7	15.3	62	21.6	21.3	-0.2	17.8	45	25.2	25.8	0.5	17.3	55
MOUNT STROMLO	G	3.5	1.1	-2.5	1.9	335	2.6	1.3	-1.3	1.9	335	4.5	4.7	0.2	6.6	333
	S	9.0	6.6	-2.4	4.0	335	8.9	6.6	-2.3	4.3	335	6.6	4.9	-1.7	1.2	333
	D	11.3	10.6	-0.8	2.4	335	16.0	14.3	-1.7	4.9	335	13.3	10.9	-2.5	3.4	333
SYOWA	V	6.3	5.4	-0.9	0.9	31	4.6	5.0	0.4	1.1	31	9.3	18.0	8.7	5.2	15
	G	1.8	1.1	-0.8	1.3	31	4.9	1.1	-3.8	1.1	31	2.7	4.7	2.0	3.8	15
	D	6.6	5.0	-1.6	4.4	31	8.6	6.9	-1.8	5.9	31	22.2	9.6	-12.6	2.5	15
TAHITI	G	1.9	1.4	-0.6	1.4	30	5.1	1.5	-3.6	1.4	171	4.4	4.5	0.1	4.2	171
	S	10.0	7.9	-2.1	23.6	30	16.1	9.5	-6.6	12.4	171	14.0	8.1	-5.9	13.9	171
	D	7.5	6.8	-0.7	2.0	30	15.1	12.9	-2.2	11.0	171	14.0	11.9	-2.1	7.6	171
HARTE	V	2.8	3.9	1.1	0.8	156	6.7	3.8	-2.9	0.7	156	10.0	8.6	-1.4	1.9	156
	S	6.9	8.6	1.7	4.0	156	6.3	6.9	0.6	4.1	156	8.4	5.1	-3.3	1.6	156
	D	4.9	7.2	2.3	2.6	156	15.0	14.0	-1.0	5.8	156	9.7	10.8	1.1	3.8	156
	G	1.3	1.2	-0.1	1.0	156	2.9	1.4	-1.5	1.1	156	2.9	3.7	0.8	3.7	156